Solutions to JEE Advanced Home Practice Test -7 |JEE 2024 | Paper-1

PHYSICS

1.(CD) The change in momentum for each time internal t_0, $\Delta p = Ft_0$ which is constant

But the change in kinetic energy in each time internal t_0 is different

First t_0 interval: $\Delta K_1 = \dfrac{\Delta p^2}{2m} = \dfrac{F^2 t_0^2}{2m}$

Second t_0 interval: $\Delta K_2 = \dfrac{(2\Delta p)^2 - (\Delta p)^2}{2m} = \dfrac{3F^2 t_0^2}{2m}$

The change in kinetic energy is $\Delta K = Fx_0 = \text{constant}$

First x_0 displacement: $\Delta p_1 = \sqrt{2m\,\Delta K_1} = \sqrt{2m\,(Fx_0)}$

Second x_0 displacement

$\Delta p_2 = \sqrt{2m(2Fx_0)} - \sqrt{2m(Fx_0)} = 0.414\sqrt{2m(Fx_0)}$

2.(ACD) Since potential energy of the particle is equal to, ϕ, x- component of the force acting on the particle is equal to

$F_x = \dfrac{-\partial\phi}{dx}$

And y-component of the force on the particle is equal to

$F_y = \dfrac{-\partial\phi}{dy}$

Hence, force acting on the particle is

$\vec{F} = -(12\hat{i} + 16\hat{j})N$

It means, force acting on the particle is constant

Hence, the particle moves with constant acceleration. So option (A) is correct.

Acceleration of the particle, $\vec{a} = \dfrac{\vec{F}}{m} = -(3\hat{i} + 4\hat{j})ms^{-2}$

Its magnitude is $a = \sqrt{3^2 + 4^2} = 5\ ms^{-2}$

Since, the particle was initially at rest at (6, 4), position of the particle at time t is given by

$x = 6 + \dfrac{1}{2}a_x t^2 = \left(6 - \dfrac{3}{2}t^2\right)m$

and $\quad y = 4 + \dfrac{1}{2}a_x t^2 = (4 - 2t^2)m$

when the particle crosses the x-axis, $y = 0$

$t_1 = \sqrt{2}\ s$

Displacement of the particle during this time, $s_1 = \dfrac{1}{2}at^2 = 5m$

Hence, work done by the force, up to this instant

$Fs_1 = 20 \times 5J = 100\ J$

Hence, option (B), is incorrect

The particle crosses y-axis when $x = 0$

Hence, $6 - \dfrac{3}{2}t_2^2 = 0$ or $t_2 = 2s$

Speed of the particle at this instant will be $v = at_2 = 5 \times 2 = 10\ ms^{-1}$

Hence, option (C) is also correct

At $t = 4\ s$, $x = 6 - \frac{3}{2}(4)^2 = -18\ m$

and $y = 4 - 2(4)^2 = -28\ m$

Hence, option (D) is also correct.

3.(AC) (A) $$x_{cm}\frac{\int xdm}{\int dm} = \frac{\int_0^L x\lambda_0\left(1+\frac{x}{L}\right)dx}{\int_0^L \lambda_0\left(1+\frac{x}{L}\right)dx} = \frac{5L}{9}$$

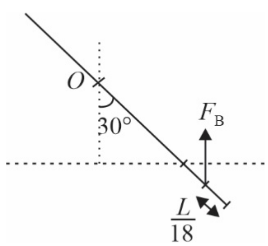
$\therefore$ form $A \Rightarrow$ distance of center of mass is

$L - \frac{5L}{9} = \frac{4L}{9}$ (A = correct)

(B) $$M_{OA} = \int dm = \int_0^L \lambda_0\left(1+\frac{x}{L}\right)dx = \frac{3\lambda_0 L}{2}$$

Or $M_{AB} = 2 \times M_0 A = 3\lambda_0 L$ (B = incorrect)

(C)



$$F_B = \left(A_0\frac{L}{9}\right)\rho g$$

$\therefore$ τ_0 due to $F_B = \left[\left[A_0\frac{L}{9}\right]\rho g\right]\left(L - \frac{L}{18}\right)\sin\theta$

Also τ_0 due to $F = FL\cos\theta$

As $\tau_0 = 0 \Rightarrow$

$$\Rightarrow \quad \left[\left(A_0\frac{L}{9}\right)\rho g\right]\frac{17}{18}L\sin\theta = F.L\cos\theta$$

$$\Rightarrow \quad \frac{17}{162}L.A_0.\rho\, g\tan 30° = F$$

Or $F = \frac{17\ LA_0.\rho g}{162\sqrt{3}}$ ($\therefore C =$ correct)

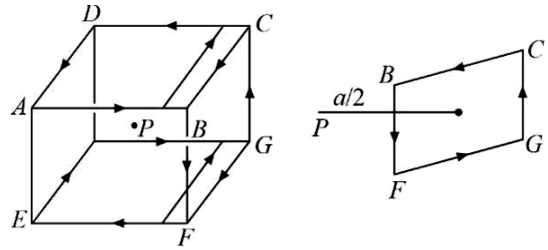
(D) Since λ is not uniform, the center of mass and center of buoyancy will be different points. (D= incorrect)

4.(BCD) The coils are in parallel, so

$$L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} \Rightarrow \int L_1\, dI_1 = \int L_2\, dI_2$$

$\Rightarrow$ Initially $I_1 = 0,\ I_2 = 0$; So, $L_1 I_1 = L_2 I_2$

5.(AD) The resultant figure can be redrawn as in figure.



The field at P due to upper and lower squares will cancel out. Resultant field at P will due to the square $BCGF$ as shown in the figure. Solve to get, $B_P = \frac{2\mu_0 I}{\pi\sqrt{3}a}$

6.(A) It is given that in the thermodynamics process amount of heat supplied is equal to the decrease in internal energy of gas thus we have in the process

$$dU = -dQ$$

Or $nC_v dT = -nCdT$

Or $C = -C_v = -\frac{R}{\gamma - 1}$ ....(i)

Now from first law of thermodynamics, we have in this process

$dQ = dU + dW$

Or $2dQ = dW$

Or $-\frac{2nR}{\gamma - 1} dT = PdV$ ....(ii)

We have differential from of gas law, $PdV + VdP = \left(\frac{1-\gamma}{2}\right) PdV$

Or $\left(\frac{1+\gamma}{2}\right) PdV = -VdP$

Or $\left(\frac{1+\gamma}{2}\right) \frac{dV}{V} = -\frac{dP}{P}$

Integrating this equation we get

$\left(\frac{1+\gamma}{2}\right) \int \frac{dV}{V} = -\int \frac{dP}{P}$; $\ln V^{\left(\frac{1+\gamma}{2}\right)} = -\ln P + C$

Or $PV^{\left(\frac{1+\gamma}{2}\right)} = \text{constant}$

As we require process equation in T and V, from gas law

$P = \frac{nRT}{V}$

Now from equation (iii)

$\left(\frac{nRT}{V}\right) V^{\left(\frac{1+\gamma}{2}\right)} = \text{constant}$

Or $TV^{\left(\frac{\gamma - 1}{2}\right)} = \text{ constant}$

7.(16.66-16.67) The formula to find component of vector a perpendicular to vector b is given by,

$$\vec{C} = \vec{a} - \frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\vec{b}$$

We know, $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

$|\vec{b}| = \sqrt{1^2 + 1^2 + 1^2}$; $\quad |\vec{b}| = \sqrt{3}$

$|\vec{b}|^2 = 3$; $\quad \vec{a} \cdot \vec{b} = 2(1) - 3(1) + 2(1)$

$\vec{a} \cdot \vec{b} = 4 - 3 = 1$

Substitute the value in the formula

$$\vec{C} = 2\hat{i} - 3\hat{j} + 2\hat{k} - \frac{1}{3} \times (\hat{i} + \hat{j} + \hat{k}),\ \vec{C} = \frac{6\hat{i} - 9\hat{j} + 6\hat{k} - \hat{i} - \hat{j} - \hat{k}}{3}$$

$$\vec{C} = \frac{5\hat{i} - 10\hat{j} + 5\hat{k}}{3}$$

$$\vec{C} = \frac{5}{3}(\hat{i} - 2\hat{j} + \hat{k}) \qquad |C| = 5\sqrt{\frac{2}{3}} \approx= \sqrt{\frac{50}{3}}$$

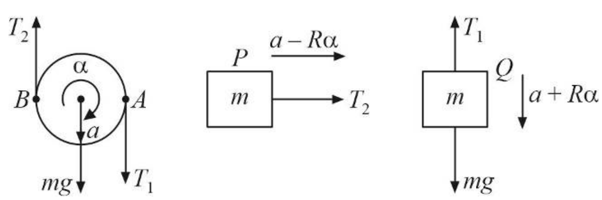
8.(2) $v = \sqrt{gh}.\dfrac{dh}{dty} = \sqrt{gh} \quad \therefore \quad t = \int_0^h \dfrac{dh}{\sqrt{gh}}$ or $t = 2\sqrt{\dfrac{h}{g}}$

Now at the time of meeting.
Time of fall of particle = time of wave

Pulse on reaching up to these $\sqrt{\dfrac{2(L-h)}{g}} = 2\sqrt{\dfrac{h}{g}} : h = \dfrac{L}{3} = 2m$

9.(0.40) Let a be the acceleration of the cylinder, then acceleration of point A on the cylinder

$$\vec{a}_A = \vec{a}_{\text{trans}} + \vec{a}_{\text{rot}}$$



$$\vec{a}_A = \vec{a}_{\text{trans}} + \vec{a}_{\text{rot}}$$

$|\vec{a}_A| = a + R\alpha = a_Q$

Similarly, the acceleration of point B on the cylinder is

$|\vec{a}_B| = a - R\alpha = a_P$

$mg - T_1 = m(a + R\alpha) = a_Q \qquad$ ......(i)

$T_1 + mg - T_2 = ma \qquad$ ......(ii)

$T_2 = m(a - R\alpha) \qquad$ ......(iii)

$T_1R + T_2R = \dfrac{mR^2}{2}\alpha \qquad$ ......(iv)

On solving these equations, we get ; $\quad a = \dfrac{2g}{3},\ \alpha = \dfrac{2g}{5R}$

10.(2.00)

At the time of maximum compression, the speeds of blocks will be the same. Let that speed be v and maximum compression be x.

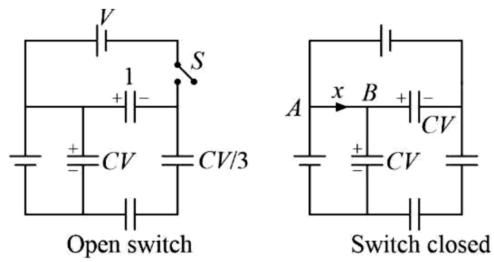
Applying conservation of momentum, $(m_1 + m_2)v = m_1v_1 + m_2v_2 \quad \Rightarrow \quad v = 4\ m/s$

Applying conservation of mechanical energy ; $\frac{1}{2}kx^2 + \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Solving, we get $x = 0.02\ m = 2\ cm$

11.(3.60)

After closing the switch capacitor between B and S changes its potential from $V/3$ to V where as other capacitor connected to point B continued to maintain its voltage at V. Let the charge flown from A to B be x then.



x = change in charge on capacitor '1' $= CV - \frac{CV}{3} = \frac{2CV}{3} = 3.60$

12.(2.5) $Y = \frac{F}{A}\frac{l}{\Delta l}$

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta l}{l} + \frac{\Delta(\Delta l)}{\Delta l} = 2\frac{\Delta d}{d} + \frac{\Delta l}{l} + \frac{\Delta(\Delta \ell)}{\Delta l} \qquad = 2\left(\frac{0.01}{1.25}\right) + \frac{0.1}{100} + \frac{0.001}{0.125} = 0.025$$

$\therefore$ Percentage error = 2.5

13.(15) The doublet forms an erect image of a reduced size. It implies that the doublet behaves as a divergent lens and the image is a virtual one.

$$m = \frac{v}{u} = \frac{I}{O} = \frac{2}{6} = \frac{1}{3} \qquad \therefore \qquad v = \frac{u}{3}$$

$u = -30\,cm$ (given)

$\therefore$ $v = -10cm$

Using, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; $\quad -\frac{1}{10} + \frac{1}{30} = \frac{1}{f} \quad \therefore \quad f = -15cm$

f being the focal length of combination of lenses A and B,

$$\frac{1}{f_A} + \frac{1}{f_B} = \frac{1}{f} = -\frac{1}{15} \qquad \ldots . \text{(i)}$$

Since, the combination is achromatic,

$$\frac{\omega_A}{f_A} + \frac{\omega_B}{f_B} = 0 \quad \text{or} \quad \frac{f_A}{f_B} = -\frac{\omega_A}{\omega_B}$$

$\therefore$ $\frac{f_A}{f_B} = -2 \quad \left[\frac{\omega_A}{\omega_B} = 2, \text{given}\right]$

OR $\quad f_A = -2f_B \qquad \ldots . \text{(ii)}$

Using Eq. (ii) in Eq. (i),

$$\frac{1}{-2f_B} + \frac{1}{f_B} = -\frac{1}{15}$$

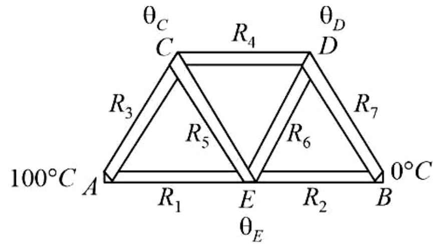
$\therefore$ $f_B = -\frac{15}{2} = -7.5\,cm$ and $f_A = -2f_B = 15cm$

Therefore, lens A is convergent with a focal length 15 cm and lens B is divergent with a focal length 7.5 cm.

14.(75.00)

This problem can be solved like electric current problem

Let $R_1, R_2, R_3, R_4, R_5, R_6$ and R_7 be the rates of heat flow through *AE*, *EB*, *AC*, *CD*, *CE*, *ED* and *DB*, respectively



Since $R_1 = R_2, \theta_E = 50°C$ (i)

$R_5 = R_6, R_3 = R_4 + R_5 = R_7$ (ii)

$R_4 + R_6 = R_7$

$$\frac{KS(\theta_C - 50)}{l} = \frac{KS}{l}(50 - \theta_D)$$

$$\theta_C + \theta_D = 100$$

$$\frac{KS}{l}(100 - \theta_C) = \frac{KS}{l}(\theta_C - 50) + \frac{KS}{l}(\theta_C - \theta_D) = \frac{KS}{l}\theta_D$$

$$4\theta_C = 250 \quad \Rightarrow \quad \theta_C = 62.5°C$$

$$\theta_D = 37.5°C \quad \therefore \quad \theta_C > \theta_E > \theta_D \quad \Rightarrow \quad \frac{\theta_E + \theta_C + \theta_D}{2} = 75°C$$

15(B). Let $F = kh^x c^y r^z$

F is force per unit area

$$[F] = [h]^x\,[c]^y\,[r]^z$$

$$[M^1 L^{-1} T^{-2}] = [M\,L^2 T^{-1}]^x [L^1 T^{-1}]^y [L]^z$$

$$\left(\text{Note} : h = \frac{\text{Energy}}{\text{frequency}}\right)$$

$\therefore \quad x = 1$

$2x + y + z = -2 \quad -x - y = -2$

Solving; $x = 1; y = 1; z = -4$

16(A). $h = 6.63 \times 10^{-27}\,\text{g}\,\text{cm}^2\,\text{s}^{-1}$

$c = 3 \times 10^{10}\,\text{cm}\,\text{s}^{-1}; \quad r = 10^{-4}\,\text{cm}$

$$\therefore \quad k = \frac{6.63 \times 10^{-27} \times 3 \times 10^{10}}{(10^{-4})^4} = 13 \times 10^{-3} \quad \therefore \quad k = \frac{13}{6.63 \times 3} \times 10^{-2} = 6.5 \times 10^{-3}$$

17.(C) Due to absorption of light in the plate, effective intensity of the source has a value smaller than the actual one. Fractional transmission through the plate can be determined as follows:

$$-dI = \mu I dx \quad \therefore \quad \frac{dI}{I} = -\mu dx$$

$$\int_{I_O}^{I} \frac{dI}{I} = -\int_{O}^{t} \mu dx; \qquad \ln\frac{I}{I_O} = -\mu t$$

$$\therefore \quad \frac{I}{I_O} = e^{-\mu t} \qquad \ldots \text{(i)}$$

Here I_O is the incident intensity and *I* is the transmitted intensity $\frac{I}{I_O}$ can be regarded as fractional transmission through the plate.

For the plate of thickness 10.2 *mm*

$$e^{-\mu t} = e^{-(0.1)(10.2)} = 0.36 \qquad [\mu = 0.1(mm)^{-1}, \text{given}]$$

Fractional transmission being 0.36, intensity of light from any source, which passes through this plate, will be effectively be 0.36 I_0.

For the plate of thickness 5.1 *mm*

$$e^{-\mu t} = e^{-(0.1)(5.1)} = 0.60$$

Thus, intensity of light passing through this plate will effectively be 0.6 I_0.

Initially, before placing any plate, S_1 and S_2 match each other when S_1 is at distance r_1 and S_2 at r_2 from the screen

$$\therefore \quad \frac{P_1}{P_2} = \frac{r_1^2}{r_2^2} \quad \ldots\ldots\text{(i)}$$

[Intensity being the same $= \frac{P_1}{r_1^2} = \frac{P_2}{r_2^2}$]

When a plate of thickness 10.2 *mm* is placed between S_1 and screen, effective power of S_1 will be $0.36P_1$. It is given that S_1 has to be moved by a distance 20 *cm* to match with S_2 again. Obviously, S_1 has to be moved towards the screen. This is because its intensity is effectively reduced so that its distance from the screen has to be shorter so as to match again with S_2.

In this case,

$$\frac{0.36P_1}{P_2} = \frac{(r_1-20)^2}{r_2^2} \quad \ldots\ldots\text{(ii)}$$

[r_1 and r_2 taken in *cm*]

Dividing Eq. (ii) by Eq. (i)

$$0.36 = \frac{(r_1-20)^2}{r_1^2} \quad \therefore \quad r_1 = 50\,cm$$

In the second situation, instead of the plate of 10.2 *mm* thickness, a plate of thickness 5.1 *mm* is kept between S_1 and screen. This makes the effective power of S_1 as $0.6P_1$. It is given that S_1 and S_2 now match each other for equal distances from the screen. It implies that $0.6P_1 = P_2$

$$\therefore \quad \frac{P_2}{P_1} = 0.6$$

Using (i) : $\frac{10}{6} = \frac{(50)^2}{r_2^2} \quad \therefore \quad r_2 = 38.7\,cm$

Hence, the correct answer is (C).

18.(B) Placing the plate of thickness 10.2 *mm* between S_1 and screen makes the power of S_1 effectively $0.36P_1$ Effective power of S_2, when plate of thickness 5.1 *mm* is placed between S_2 and screen, will be $0.6P_2$. In this situation, if r_1 and r_2 are respectively, the distance of S_1 and S_2 from the screen so that they match each other, $\frac{0.36P_1}{0.6P_2} = \frac{r_1^2}{r_2^2}$

As $\frac{P_1}{P_2} = \frac{10}{6}$, $\quad \therefore \quad \frac{r_1^2}{r_2^2} = 1 \quad \therefore \quad \frac{r_1}{r_2} = 1$

Therefore, (B) is the correct option.

CHEMISTRY

1.(CD) $2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2 + O_2$

$2LiNO_3 \xrightarrow{\Delta} Li_2O + 2NO_2 + \frac{1}{2}O_2$

2.(ACD)

As the M – C π bonding increases the C – O bond length increases

As the electron density on the central metal atom increases, the C – O bond length increases

In $Cr(CO)_6$

EAN = 24 + 12 = 36

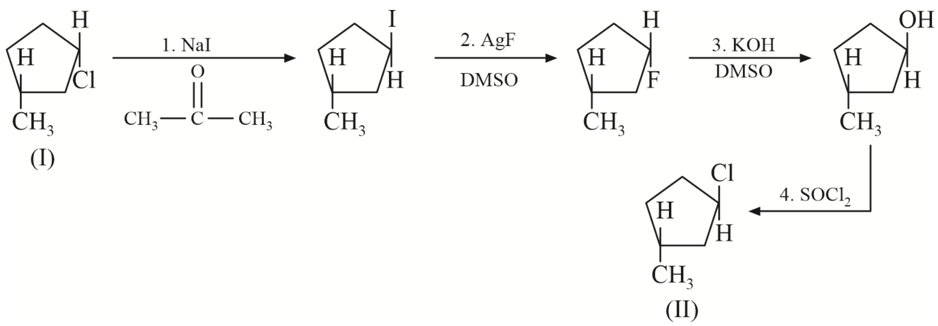
3.(BC) Nitrogen being second period element, d-orbitals are not available for bonding

$4H_3PO_3 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$

The reducing nature of hydrides increases down the group

The basic nature of oxides increases down the group.

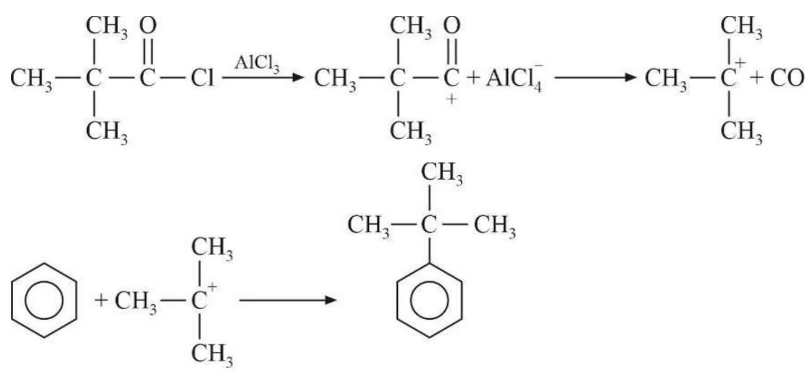
4.(AB)



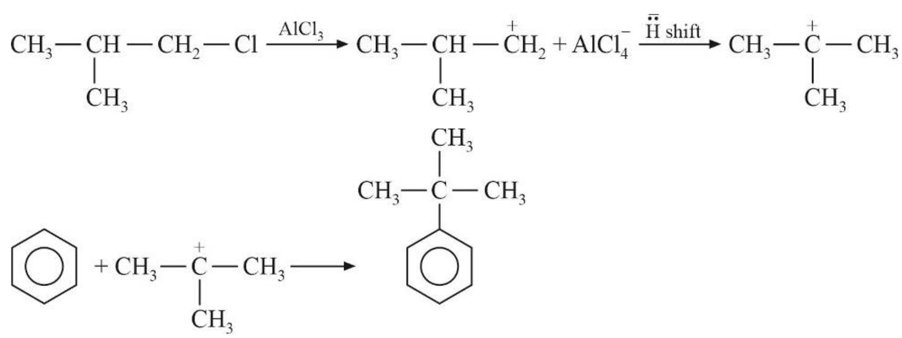
(I) and (II) are geometrical isomers and diastereomers.

5.(ABCD)

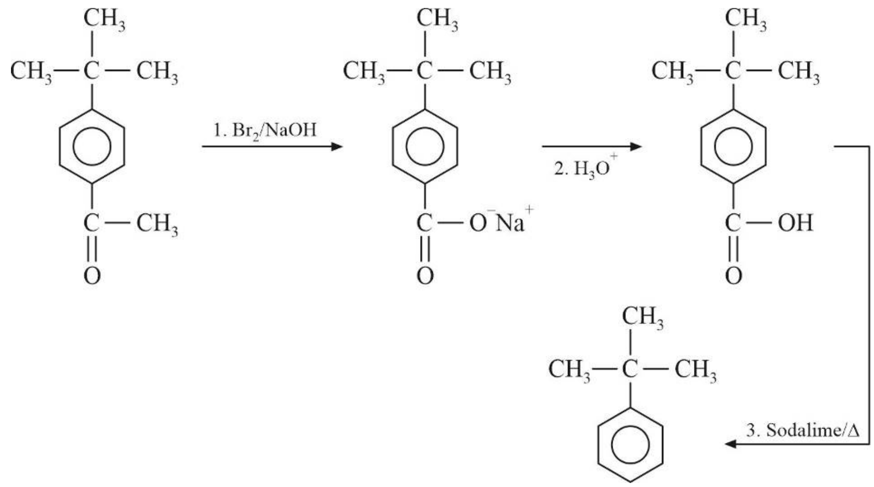
(A)



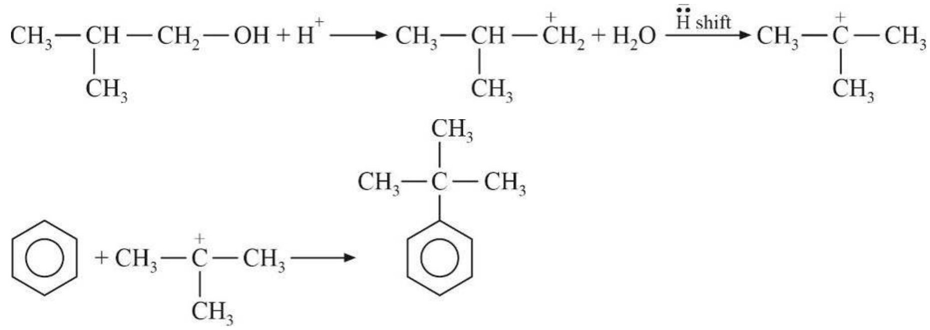
(B)



(C)



(D) $H_3PO_4 \longrightarrow H_2PO_4^- + H^+$



6.(BD) At point a,

$$P_0V_0 = 1 \times RT_0 \qquad \Rightarrow \qquad T_0 = \frac{P_0V_0}{R}$$

At point b,

$$P_0 \times 4V_0 = 1 \times R \times T_b \qquad \Rightarrow \qquad T_b = \frac{4P_0V_0}{R} = 4T_0$$

At point c,

$$2P_0 \times 4V_0 = 1 \times R \times T_c \qquad \Rightarrow \qquad T_c = \frac{8P_0V_0}{R} = 8T_0$$

$$\Delta U_{a\to b} = 1 \times \frac{3}{2} R \times (4T_0 - T_0) = \frac{9}{2} RT_0$$

$$\Delta U_{b\to c} = 1 \times \frac{3}{2} R \times (8T_0 - 4T_0) = 6\ RT_0$$

$$\Delta U_{a\to c} = \Delta U_{a\to b} + \Delta U_{b\to c} \qquad = \frac{9}{2} RT_0 + 6RT_0$$

$$\Delta U_{a\to c} = \frac{21}{2} RT_0; \qquad W_{a\to b} = -P_0 \times (4V_0 - V_0) = -3P_0V_0 \qquad = -3RT_0$$

$$\Delta U_{a\to b} = q_{a\to b} + W_{a\to b}; \qquad \frac{9}{2} RT_0 = q_{a\to b} - 3RT_0$$

$$q_{a\to b} = \frac{9}{2} RT_0 + 3RT_0 = \frac{15}{2} RT_0$$

Process $a \to b$ is isobaric

Hence $\qquad q_{a\to b} = \Delta H_{a\to b}; \qquad \Delta H_{a\to b} = \frac{15}{2} RT_0$

7.(5) Paramagnetic species $\rightarrow O_2,\ O_2[AsF_6],\ KO_2,\ [Cu(NH_3)_4]^{2+},$ and $(NH_4)_2[CoCl_4]$

8.(3106)

Number of moles of $(NH_4)_2SO_4 = \frac{1584}{132} = 12$

$$\underset{\substack{12 \\ 0}}{(NH_4)_2SO_4} + Ca(OH)_2 \longrightarrow \underset{\substack{0 \\ 12}}{CaSO_4 \cdot 2H_2O} + \underset{\substack{0 \\ 24}}{2NH_3}$$

Moles of NH_3 formed = 24

Number of moles of $CrCl_3 \cdot 6H_2O = \frac{1066}{266.5} = 4$

$$\underset{\substack{4 \\ 0}}{CrCl_3 \cdot 6H_2O} + \underset{\substack{24 \\ 0}}{6NH_3} \longrightarrow \underset{\substack{0 \\ 4}}{[Cr(NH_3)_6]Cl_3} + 6H_2O$$

Mass of gypsum = 12 × 172 = 2064 gram

Mass of $[Cr(NH_3)_6]Cl_3 = 4 \times 260.5 = 1042$ gram

Total mass of gypsum and $[Cr(NH_3)_6]Cl_3$

= 2064 + 1042 = 3106 gram

9.(155)

$$d = \frac{z \times M}{a^3 \times N_A}; \quad z = 4 \times \frac{99.9}{100} = 3.996$$

$$d = \frac{3.996 \times 40}{(0.556 \times 10^{-7})^3 \times 6 \times 10^{23}}; \quad d = 1.55 \text{ gram/cm}^3$$

Hence 100 d = 155

10.(148)

$$E^\circ = \frac{0.0591}{n} \log K$$

$$\underset{\substack{1 \\ 1-y}}{Fe + Zn^{2+}} \rightleftharpoons \underset{\substack{0 \\ y}}{Fe^{2+}} + Zn$$

$$E^\circ = -0.32 = \frac{0.0591}{2} \log \frac{y}{(1-y)}$$

$\Rightarrow \quad y = 1.48 \times 10^{-11} \Rightarrow \quad [Fe^{2+}] = 1.48 \times 10^{-11}$

$= 148 \times 10^{-13} = x \times 10^{-13} \quad \Rightarrow \quad x = 148$

11.(150)

$\Delta U = q + W$

$W = \Delta U - q$

$= 353 - 5005$

$W = -4652$ J

$$W = -\int PdV = -\int_1^{V_2} \frac{20}{V} dV$$

$W = -20 \ln V_2$

$W = -20 \times 2.303 \log V_2$ litre-atm

$W = -20 \times 2.303 \times 101.325 \log V_2$ Joule

$\Rightarrow \quad -4652 = -20 \times 2.303 \times 101 \log V_2 \quad \Rightarrow \quad V_2 = 10$ litre

Hence, $15V_2 = 150$

12.(40) $P_s = x_A P_A^\circ + x_B P_B^\circ$; $\quad 100 = x_A \times 200 + (1 - x_A) \times 75$

$\Rightarrow \quad x_A = 0.2$; $\quad y_A = \dfrac{x_A \times P_A^\circ}{P_s}$

$y_A = \dfrac{0.2 \times 200}{100}$; $\quad y_A = 0.4$

Mole percent of A $= y_A \times 100 = 0.4 \times 100 = 40\%$

13.(50) $\underset{}{MgF_{2(s)}} \rightleftharpoons \underset{s}{Mg^{2+}} + \underset{2s}{2F^-}$, K_{sp}

$F^- \quad + \quad H^+ \rightleftharpoons HF$, $\dfrac{1}{K_a}$

$2s \qquad 10^{-4}$

$\left(\dfrac{K_{sp}}{s}\right)^{1/2} \qquad 10^{-4} \qquad 2s$

$\dfrac{1}{K_a} = \dfrac{2s}{10^{-4} \times \left(\dfrac{K_{sp}}{s}\right)^{1/2}}$; $\quad \dfrac{1}{3.5} \times 10^4 = \dfrac{2s^{3/2}}{10^{-4} \times (K_{sp})^{1/2}}$

$s^{3/2} = \dfrac{1}{7} \times (K_{sp})^{1/2}$; $\quad s^3 = \dfrac{1}{49} \times K_{sp}$

$s^3 = \dfrac{1}{49} \times \dfrac{9}{14} \times 10^{-8}$; $\quad s^3 = \dfrac{1}{(7)^3} \times (45) \times 10^{-9}$

$s = 5 \times 10^{-4}$; $\quad s = 50 \times 10^{-5}$ mol/litre $= Y \times 10^{-5} \quad \Rightarrow \quad$ Hence Y = 50

14.(30) For solute S

$\Delta T_b = i \times 0.1 \times K_b$

For other solute

$\Delta T_b = 1 \times 0.08 \times K_b$

Hence, $i \times 0.1 \times K_b = 0.08 \times K_b$

$\Rightarrow \quad i = 0.8$

$3S \rightleftharpoons S_3$

$a \qquad 0$

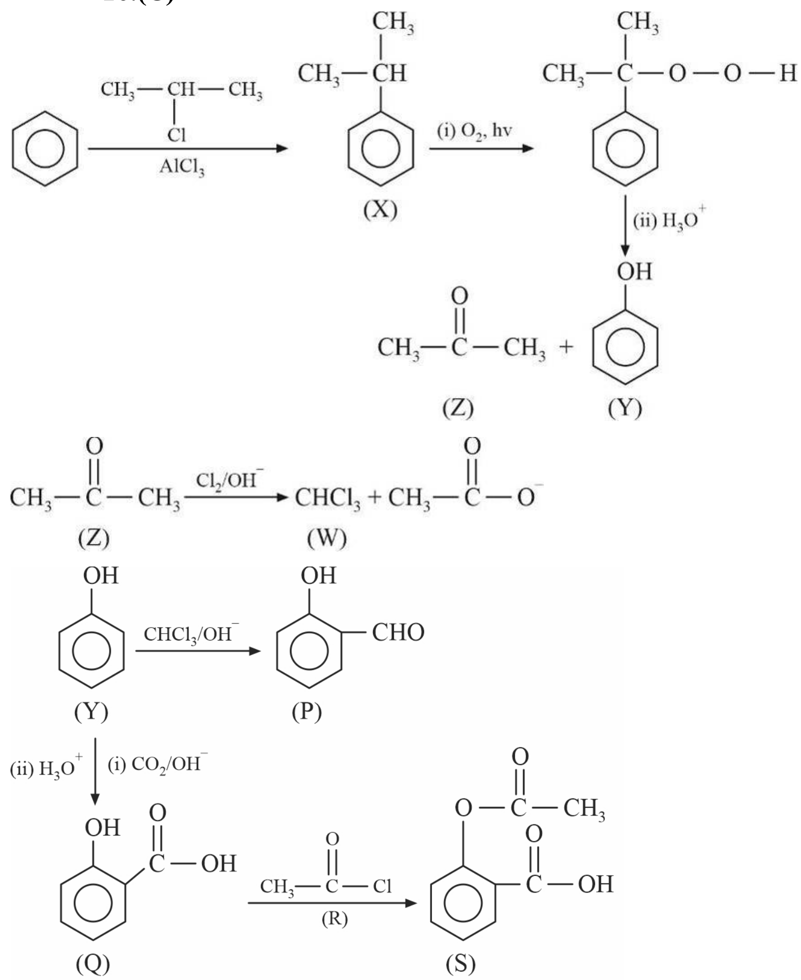
$a(1-\alpha) \qquad a\alpha/3$

$i = \dfrac{a(1-\alpha) + \dfrac{a\alpha}{3}}{a}$; $\quad i = 1 - \dfrac{2\alpha}{3}$

$0.8 = 1 - \dfrac{2\alpha}{3}$

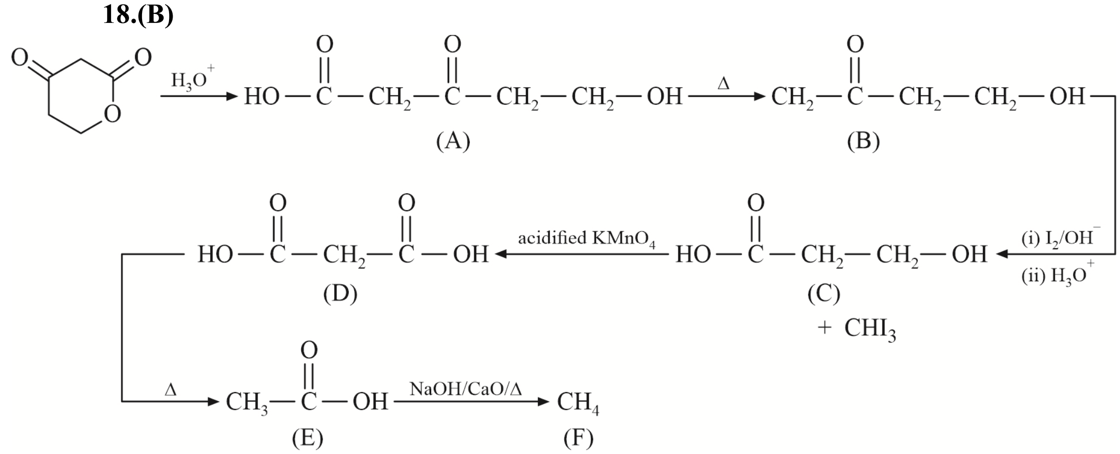
$\dfrac{2\alpha}{3} = 0.2 \Rightarrow \alpha = 0.3 = 30 \times 10^{-2} \quad \Rightarrow \quad Y = 30$

15.(A) **16.(C)**



(Aspirin, Used as Pain Killer)

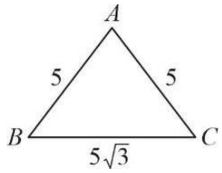
17.(D) **18.(B)**



MATHEMATICS

1.(ABD)

2.(AB) Given that $a = 5\sqrt{3},\ b = 5,\ c = 5$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 5^2 - (5\sqrt{3})^2}{2 \times 5 \times 5}$$

$\Rightarrow \qquad \cos A = -\frac{25}{50} = -\frac{1}{2}$

$A = 120°$

Area of $\Delta ABC \ = \frac{1}{2} bc \sin A = \frac{1}{2} \times 5 \times 5 \times \sin 120° = \frac{1}{2} \times 5 \times 5 \times \frac{\sqrt{3}}{2} = \frac{25\sqrt{3}}{4}$

Radius of incircle of $\Delta ABC \qquad = r = \frac{\Delta}{s} = \frac{\frac{25\sqrt{3}}{4}}{\frac{10 + 5\sqrt{3}}{2}}$

$$= \frac{5}{2}(2\sqrt{3} - 3)$$

Radius of circumcircle of $\Delta ABC = R$

$$= \frac{abc}{4\Delta} = \frac{5\sqrt{3} \times 5 \times 5}{4 \times \frac{25\sqrt{3}}{4}} = 5\ cm$$

3.(BCD)

(A) $\qquad \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = -\hat{i} + 3\hat{j} + 5\hat{k}$

equation of plane is

$-1(x - 3) + 3(y - 2) + 5(z + 1) = 0$

$-x + 3y + 5z + 2 = 0$

$x - 3y - 5z = 2$

(B) $\qquad \cos\theta = \left|\frac{2 - 2 - 1}{\sqrt{6}\ \sqrt{6}}\right| = \frac{1}{6}$

(D) $\qquad$ Any plane passing through the intersection of P_1 and P_2 is

$(2x - y + z - 2) + \lambda(x + 2y - z - 3) = 0$

(1, 3, –2) lies on it, therefore $\lambda = \frac{5}{6}$

equation of required plane is

$(2x - y + z - 2) + \frac{5}{6}(x + 2y - z - 3) = 0 \qquad \Rightarrow \qquad 17x + 4y + z - 27 = 0$

4.(ABC)

(A) $\lim_{x\to 0} f(x^2) = \lim_{x\to 0}(f(x))^2 \quad \Rightarrow \quad f(0)=1$

Continuity forces $f(x)=1$ for all x

(B) If $f(a)=0 \quad \Rightarrow \quad f(a^{2^{-n}}) = (f(a))^{2^{-n}}$

$\Rightarrow \quad f(1)=0$

Assume for some b, $f(b)>0$

$\Rightarrow \quad f(b^{2^{-n}}) = (f(b))^{2^{-n}} \quad \Rightarrow \quad f(1)=1$ contradiction

(C) Since all powers of x work, we can splice suitable powers at $x = 1$ to get the total integral less than 1

Define $f(x)\begin{Bmatrix} x^{m+1} & x\in(0,1) \\ 1 & x=1 \\ x^{-(n+2)} & x\in(1,\infty) \end{Bmatrix}$

Where m, n are any positive integers.

$$\int_0^\infty f(x)dx = \frac{1}{m+2}+\frac{1}{n+1} \le \frac{1}{3}+\frac{1}{2} \le \frac{5}{6}$$

Infinite choices for m and n.

5.(ABCD)

$f(f(x))=x \quad \Rightarrow \quad f(f(0))=0 \quad \Rightarrow \quad f(1)=0$

$f:[0,1]\to[0,1]$ is invertible $\quad \Rightarrow \quad f$ is decreasing

Let $\quad I=\int_0^1 (x-f(x))^{2020}dx$, put $f(x)=t \quad \Rightarrow \quad I=\frac{1}{2021}$

6.(BC)

Notice $f(0) = 0$

and $\quad f'(x)=2f(x) \quad \Rightarrow \quad f(x)=0 \quad \forall\, x$

7.(1) $\left(\log_3 7\right)^{8\log(\log_9 49)\,7} \times 5^{-8\log_5 7}$

$\left(\log_3 7\right)^{8\log(\log_3 7)\,7} \times 5^{\log_5 7^{-8}}$

$\left(\log_3 7\right)^{\log(\log_3 7)\,7^8} \times 7^{-8} = 7^8 \times 7^{-8} = 1$

8.(216) Sum of all digits in set is 0 + 1 + 2 + 3 + 4 + 5 = 15

Case I

If number is formed using 1, 2, 3, 4, 5 then number of numbers = 5! = 120

Case II

If number is formed using 0, 1, 2, 4, 5 then number of numbers = 4 × 4! = 96

So, total numbers = 120 + 96 = 216

9.(3638)

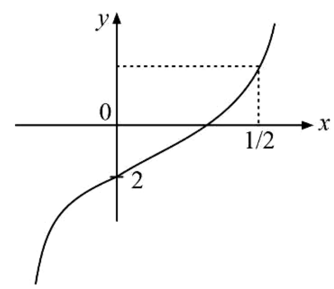
Common elements in X and Y are 18, 38, 58, 78, …… 8078

Number of common elements in X and Y is 404

So, Number of elements in XUY is

2021 + 2021 – 404 = 3638

10.(1) $\cos^{-1}\left(\frac{-x/2}{1+\frac{x}{2}}+\frac{x}{1+x}\right)=\cos^{-1}\left(\frac{x^2}{1-x}-\frac{x\times\frac{x}{2}}{1-\frac{x}{2}}\right)$



$\Rightarrow \quad \frac{-x}{2+x}+\frac{x}{1+x}=\frac{x^2}{1-x}-\frac{x^2}{2-x}$

$\boxed{x=0}$

$\Rightarrow \quad \underbrace{\frac{1}{1+x}-\frac{1}{2+x}=\frac{x}{1-x}-\frac{x}{2-x}}_{\Downarrow}$

$\Rightarrow \quad \underbrace{x^3+2x^2+5x-2}_{y}=0$

$\Rightarrow \quad y'=\underbrace{3x^2+4x+5}_{a>0 \text{ and } D<0}$

$\Downarrow$

$y'>0$

and $\quad y\left(\frac{1}{2}\right)=+ve$

$\Downarrow$

There exist a unique solution in $\left(0,\frac{1}{2}\right)$

11.(5) $P_n=\left(\left(\frac{n+2}{n}\right)\left(\frac{n+4}{n}\right)\left(\frac{n+6}{n}\right)\ldots\ldots\left(\frac{n+2n}{n}\right)\right)^{\frac{1}{n}}$

$\Rightarrow \quad P_n=e^{\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{2r}{n}\right)}$

$\Rightarrow \quad L=e^{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{2r}{n}\right)}$

$\Rightarrow \quad L=e^{\int_0^1 \ln(1+2x)dx} \rightarrow I \qquad \Rightarrow \qquad I=\int_0^1 \underbrace{1.}_{II}\ \underbrace{\ln(1+2x)}_{I}dx$

$I=\ln(1+2x)\times x\Big|_0^1-\int_0^1\frac{1\times 2x}{2x+1}dx$

$I=\frac{3}{2}\ln 3-1$

$\therefore \quad L=e^{\frac{3}{2}\ln 3-1}$

$L=\frac{\sqrt{27}}{e}$

$\Rightarrow \quad Le=\sqrt{27}$

$\Downarrow$

$[Le]=5$

12.(5) $\vec{a}\cdot\vec{c} = |\vec{a}||\vec{c}|\cos\beta$

$\vec{a}\cdot\vec{c} = 3\cos\beta$

$\vec{b}\cdot\vec{c} = 3\cos\beta$

$\Rightarrow \quad \vec{c} = m\vec{a} + n\vec{b} + \vec{a}\times\vec{b}$

$\vec{a}\cdot\vec{c} = m = 3\cos\beta$

$\vec{b}\cdot\vec{c} = n = 3\cos\beta$

$\Rightarrow$ Let $m = n = 3\cos\beta = \lambda$ $\quad\Rightarrow\quad \vec{c} = \lambda\vec{a} + \lambda\vec{b} + \vec{a}\times\vec{b}$

$\Rightarrow \quad |\vec{c}|^2 = \lambda^2 + \lambda^2 + |\vec{a}\times\vec{b}|^2 = 9 \quad\Rightarrow\quad 2\lambda^2 + 1 = 9$

$\Rightarrow \quad \lambda^2 = 4$

$\Downarrow$

$9\cos^2\beta = 4$

13.(2) $\sqrt{3}p\sin x + 2q\cos x = r$

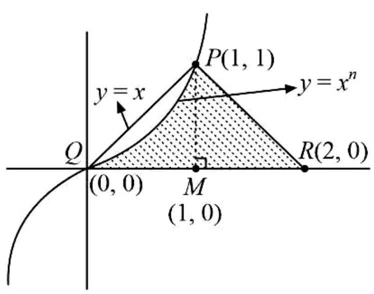
$\Rightarrow \quad \sqrt{3}p\sin\alpha + 2q\cos\alpha = r$

$\underline{\sqrt{3}p\sin\beta + 2q\cos\beta = r}$

$\Rightarrow \quad \sqrt{3}p(\sin\alpha - \sin\beta) + 2q(\cos\alpha - \cos\beta) = 0 \quad\Rightarrow\quad \sqrt{3}p\cos\frac{\alpha+\beta}{2} = 2q\sin\frac{\alpha+\beta}{2}$

$\Rightarrow \quad \frac{p}{q} = \frac{2}{\sqrt{3}}\tan\frac{\alpha+\beta}{2} \quad\Rightarrow\quad \frac{p}{q} = \frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{3}}$

14.(9)



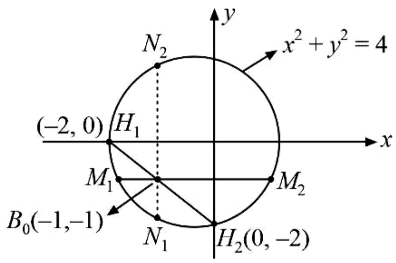
$\Rightarrow \quad \Delta PQR$ is right angled at P $\quad\Rightarrow\quad A_1 = \frac{1}{2}\times PQ\times PR = \frac{1}{2}\times\sqrt{2}\times\sqrt{2} = 1$

$\Rightarrow$ Area of shaded portion $= 0.6\times A_1$

$\Rightarrow \quad \int_0^1 x^n dx + \frac{1}{2}\times 1\times 1 = 0.6$

$\Rightarrow \quad \frac{1}{n+1} = 0.6 - 0.5 \quad\Rightarrow\quad \frac{1}{n+1} = \frac{1}{10} \quad\Rightarrow\quad n = 9$

15.(C)



Equation of chord of contact to a circle drawn from (h, k) is:

$hx + ky = 4$

(1) Equation of $M_1M_2 : y = -1 \quad \Rightarrow \quad M_3(0,-4)$

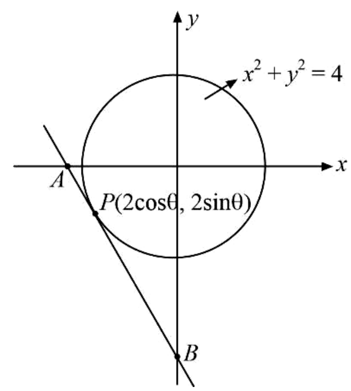
(2) Equation of $N_1N_2 : x = -1 \quad \Rightarrow \quad N_3(-4, 0)$

(3) Equation of $H_1H_2 : y + 1 = -(x+1)$

$x + y = -2 \quad \Rightarrow \quad H_3(-2,-2)$

16.(D) Tangent at 'P' is given by

$x\cos\theta + y\sin\theta = 2$



$\Rightarrow \quad A\left(\frac{2}{\cos\theta}, 0\right), B\left(0, \frac{2}{\sin\theta}\right)$

$\Rightarrow$ Let 'M' is midpoint of AB and let M be (h, k)

$$\Rightarrow \quad h = \frac{\frac{2}{\cos\theta} + 0}{2},\ k = \frac{0 + \frac{2}{\sin\theta}}{2} \quad \Rightarrow \quad \cos\theta = \frac{1}{h},\ \sin\theta = \frac{1}{k}$$

$$\Rightarrow \quad \cos^2\theta + \sin^2\theta = \frac{1}{h^2} + \frac{1}{k^2} \quad \Rightarrow \quad x^2 + y^2 = x^2y^2$$

17.(A) $\text{Probability} = \dfrac{\text{Number of favourable arrangements}}{\text{Total number of arrangements}}$

$= \dfrac{D_3}{4!} \quad \rightarrow$ De-arranging of 3 objects

$$D_3 = 3!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}\right) \quad \Rightarrow \quad \frac{2}{24} = \frac{1}{12}; \qquad D_3 = 2$$

18.(A) Number of total arrangements = 4!

Number of arrangements favouring $E_2 \cap E_3 \cap E_4$:

$\Rightarrow$ constraints of P_1 and P_4 are same

$\Rightarrow$ $\underline{P_1}\ \underline{P_3}\ \_\ \_ \Rightarrow$ No such arrangement is possible

$\underline{P_1}\ \underline{P_4}\ \underline{P_2}\ \underline{P_3} \Rightarrow$ No such that arrangement is possible

There are

$\Rightarrow$ No such arrangement where P_1, P_4 occupies 1st position

Constraints of P_2 and P_3 are same

$\rightarrow$ $\underline{P_2}\ \underline{P_4}\ \underline{P_1}\ P_3$ ✓

$\rightarrow$ $\underline{P_3}\ \underline{P_1}\ \underline{P_4}\ \underline{P_2}$ ✓

$\therefore$ Probability $(E_2 \cap E_3 \cap E_4) = \dfrac{2}{4!} = \dfrac{1}{12}$